

# A MARKOV CHAIN MODEL FOR PHILIPPINE RAINFALL DATA\*

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## 1. INTRODUCTION

### 1.1 Significance of the Study

Rainfall or the lack of it at particular times plays an important role in the lives and activities of man. Its effects could either be detrimental or beneficial. The presence or absence of rainfall at crucial times determines the success or failure of many agriculture and nonagricultural operations. Specific situations in which rainfall is a dominant factor are: 1) germination of seeds, b) disease susceptibility in periods of plant growth, c) fertilizer, insecticides and herbicides application, d) concrete pouring in building construction, and e) scheduling of outdoor activities, e.g. sports.

The relationship between rainfall and yield of some grain crops has attracted the interest of a number of researchers as early as 1924 when R.A. Fisher developed a special statistical technique for examining the effect of rainfall on the annual yield of wheat at Rothamstead. Gangopadhyaya and Sarker (1965) and Sreenivasan and Banerjee (1973) utilized the: "Fisher's response curve" to study the effect of rainfall distribution on the yield of wheat crop at the different crop weather observatory stations in India. Dunlop (1970) also discussed various aspects of the increasing impact of reliable, long range weather forecasting on recreational activities which include the economic repercussions of inadequate forecasting on

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\*Paper presented at the Third National Convention on Convention on Statistics held on 13 and 14 December 1982 at the Philippine International Convention Center.

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recreation centers, the need to chart geographically the resort areas that should be covered with more complex forecasts, and the desirability of extended services to fulfill the special needs of recreational pursuits strongly affected by meteorological factors like rain.

A knowledge therefore of the probability of occurrence of rainfall at particular times is of great importance in the decision making process and management of a number of undertakings.

### *1.2 Objectives of the Study*

If it has been said that day-to-day rainfall fluctuation exists, it must also be recognized that rainfall follows a certain pattern and it is this persistent pattern which is of greater interest than fluctuation that motivated the author to study Philippine rainfall. The aims of this paper can be summed up into one major objective, namely: To determine whether a Markov chain probability model fits well the Philippine rainfall patterns. Specifically, it involves the following:

1. Determining the probability that a given day of the year will be dry or wet.
2. Determining the probability that a given sequence of wet and dry days will occur at a particular time of the year.
3. Comparing the goodness of fit between a simple and a second order Markov chain model.

### *1.3 Conceptual Framework of the Study*

#### *1.3.1 Multiplication law of probability and Markov chains*

Consider a sequence of  $(n+1)$  consecutive days beginning with the  $(t)^{\text{th}}$  day of the year and ending with the  $(t+n)^{\text{th}}$  of the year. For our purpose, March 1 will correspond to  $t=1$ ; henceforth, February 28 will correspond to  $t=365$ . For a given  $t$ , let  $X_{t+i}$  ( $i = 0, 1, \dots, n < 365$ ) where  $X_{t+1} = D$  or  $W$ ;  $D$  for dry and  $W$  for wet.

Let  $P(X_t, X_{t+1}, \dots, X_{t+n})$  denote the probability that the particular sequence in parenthesis will occur. For a given  $t$  and  $n$ , we assume such a probability is constant over years and if records of 50 years is available, these would provide a

random sample of 50 independent observations to estimate such probabilities.

One simple point estimate for the probability of such occurrence is to consider its relative frequency. It is obtained by taking the ratio of the number of years in which such sequence occurred to the number of years of record. However, the estimation of such probabilities becomes unmanageable as  $n$  becomes larger.

An alternative approach is to consider the following multiplication law in probability theory.

$$P(X_t, X_{t+1}, \dots, X_{t+n}) = P(X_t) \cdot P(X_{t+1} | X_t) \cdot P(X_{t+2} | X_t, X_{t+1}) \dots P(X_{t+n} | X_t, \dots, X_{t+n-1}) \quad (1)$$

For example,

$$P(D_5, D_6, W_7) = P(D_5) \cdot P(D_6 | D_5) \cdot P(W_7 | D_5, D_6)$$

where  $P(D_5)$  = probability that the 5<sup>th</sup> day will be dry.  
= initial probability of the sequence

$P(D_6 | D_5)$  = probability that the 6<sup>th</sup> day will be dry given that the 5<sup>th</sup> day or the preceding day was dry.

$P(W_7 | D_5, D_6)$  = probability that the 7<sup>th</sup> day will be wet given that the 5<sup>th</sup> and the 6<sup>th</sup> days were dry.

Under certain conditions, if observations from one day to another are stochastically independent, then the right hand-side of (1) can be simplified in the following manner:

$$P(X_t, X_{t+1}, \dots, X_{t+n}) = P(X_t) \cdot P(X_{t+1}) \dots P(X_{t+n})$$

However this assumption in most cases does not hold true. A more logical assumption would be consider that present condition is dependent on the preceding condition. Then we have the following:

$$P(X_t, X_{t+1}, \dots, X_{t+n}) = P(X_t) \cdot P(X_{t+1} | X_t) \cdot P(X_{t+2} | X_{t+1}) \dots P(X_{t+n} | X_{t+n-1}). \quad (2)$$

$$\text{or } P(X_t, X_{t+1}, \dots, X_{t+n}) = P(X_t) \cdot P(X_{t+1} | X_t) \cdot P(X_{t+2} | X_t, X_{t+1}) \dots P(X_{t+n} | X_{t+n-2}, X_{t+n-1}) \quad (3)$$

(2) is called a first order or simple Markov chain model while (3) is a second order Markov chain model. The order of the model however can also be extended to greater than two. The probabilities on the right hand sides of (2) and (3) are called transition probabilities as they denote the probability of moving from one state to another (Feller, 1950).

### 1.3.2 The Model

A two-state Markov chain will be the model for the sequences of wet and dry days. For purposes of convenience, let the states be  $i = 1, 2$ ; (1=dry; 2=wet) and the times of observations be  $t=1, 2, \dots, 365$ . Let  $P_{ij}(t)$  ( $i, j = 1, 2$ ;  $t = 1, 2, \dots, 365$ ) be the probability of state  $j$  at time  $t$  given state  $i$  at time  $t-1$ . We assume that of the  $n$  years of available record, there are  $n_i(1)$  in state  $i$  at time  $t=1$ . An observation on a given day consists of the sequence of state the day is in at  $t = 1, 2, \dots, 365$ , namely  $i(1), i(2), \dots, i(365)$ . Given the initial states  $i(1)$ , there are  $(2)^{364}$  possible sequences. These represent mutually exclusive events with probabilities.

$$P_{i(1)i(2)}(2) P_{i(2)i(3)}(3) \dots P_{i(364)i(365)}(365) \quad (4)$$

In this case the transition probabilities  $P_{ij}(t)$  are assumed not stationary.

Let  $n_{ij}(t)$  denote the number of years (observations) in state  $i$  at  $(t-1)$  and  $j$  at  $t$ . The set of  $n_{ij}(t)$ , ( $i, j = 1, 2$ ;  $t=2, \dots, 365$ ) a set of  $(2^2)(364)$  numbers forms a set of sufficient statistics for the observed sequences. To show this, let  $n_{i(1)i(2)} \dots i(365)$  be the number of years whose sequence of state is  $i(1), i(2), \dots, i(365)$ .

Then

$$n_{gj}(t) = \sum n_{i(1)i(2) \dots i(365)} \quad (5)$$

where the sum is over all values of the  $i$ 's with  $i(t-1) = g$  and  $i(t) = j$ . The probability, in the  $(n)(2)(364)$  dimensional space describing all sequences for all  $n$  years of record (for each initial state, there are  $n(364)$  dimension) of a given ordered set of sequences for the  $n$  years is

$$\begin{aligned} & \pi [P_{i(1)i(2)}^{(2)} P_{i(2)i(3)}^{(3)} \dots P_{i(364)i(365)}^{(365)}]^{n_{i(1)i(2) \dots i(365)}} \\ & (365) \\ & = (\pi [P_{i(1)i(2)}^{(2)}]^{n_{i(1)i(2) \dots i(365)}}) \dots (\pi [P_{i(364)i(365)}^{(365)}]^{n_{i(1)i(2) \dots i(365)}}) \\ & = (\pi P_{i(1)i(2)}^{(2) n_{i(1)i(2)}})^{(2)} \dots (\pi P_{i(364)i(365)}^{(365) n_{i(364)i(365)}})^{(365)} i(1)mi(2) \dots, i(364), i(365) \\ & = \prod_{t=2}^{365} \prod_{g,j} \pi p_{gj}(t)^n g_j(t) \quad (6) \end{aligned}$$

where the products in the first two lines are over all values of (365) indices. Thus, the set of numbers  $n_{ij}(t)$  is (6) multiplied by an appropriate function of factorials. Let  $n_i(t-1) = \sum_{j=1}^2 n_{ij}(t)$ . Then the conditional distribution of  $n_{ij}(t)$ ,  $j = 1, 2$ , given  $n_i(-1)$  is

$$\frac{n_i(t-1)!}{\prod_{j=1}^2 n_{ij}(t)!} \prod_{j=1}^2 \pi p_{ij}(t)^{n_{ij}(t)}$$

The distribution of the  $n_{ij}(t)$  (conditional on the  $n_i(1)$ ) is

$$\prod_{t=2}^{365} \left[ \prod_{i=1}^2 \left[ \frac{n_i(t-1)!}{\prod_{j=1}^2 n_{ij}(t)!} \prod_{j=1}^2 \pi p_{ij}(t)^{n_{ij}(t)} \right] \right]$$

Thus  $n_{ij}(t)$  are minimal set of sufficient statistics for the transition probabilities  $P_{ij}(t)$  (Anderson and Goodman, 1957).

#### 1.4 Review of Related Literature

Studies involving estimating probability distribution of length of sequence of dry and wet days have received attention from a number of researchers during the past half century. Some of the more popular probability distributions utilized to fit rainfall data are the gamma distribution, log-normal distribution and the Markov chain model. The Markov chain model, however, has not always been successful as compared with the other two, i.e., in some cases the fit is not good. This could be attributed to the fact that fitting was limited only to the first order model. Gabriel and Neumann (1962) were probably the first researchers to make a formal study on the application of Markov processes on rainfall patterns. They found out that the Markov chain probability model fitted Tel-Aviv data of daily rainfall occurrence. They obtained the distribution of the number of rainy days per week, month or any other period. However, the number of rainy days in different months are apparently independent.

Caskey (1963) and Weiss (1964) had shown that the Markov chain probability model fitted sequences of wet or dry days in records of various lengths and for several climatically different areas in the United States.

Wiser (1965) proposed a number of modifications to the Markov chain model for cases in which the simple model does not fit sequences of wet or dry days. The modified models were shown to fit the observed records in most cases, and can also be applied to sequences of wet or dry hours.

Feyerharm and Bark (1965) also developed procedures to estimate the probability of occurrence for a given consecutive sequences of wet or dry days which begins with any day of the year. The procedure suggested that the probabilities:

$P(D_t)$  = probability that the  $t^{th}$  day of the year will be dry and

$P(D_t | D_{t-1})$  = probability that the  $t^{th}$  day of the year will be dry given that  $(t-1)^{st}$  day is dry.

may be estimated by fitting partial sums of Fourier series to relative frequencies computed for each day of the year. From these results the other probabilities like  $P(W_t)$ ,  $P(W_t | D_{t-1})$ ,  $P(D_t | W_{t-1})$  and  $P(W_t | W_{t-1})$  can be estimated since they are related to the first two. Such probabilities evaluated for each day of the year suffice to estimate the probability of occurrence of any sequences of wet and dry days under the assumption that such sequence can be described by a Markov chain of order one.

Bayne and Weber (1973) also applied the probability model for estimating the probability of a sequence of wet and dry days developed by Feyerharm and Bark to rainfall data from forty-eight stations in North Carolina. The model estimates the probability of a sequence of dry and wet days on the basis of a first-order Markov theory. They presented equations for estimating initial and transitional probabilities based on frequency estimators and an equation for reducing the variance of a frequency estimator by taking into account seasonal variation.

## 2. METHODOLOGY

### 2.1 Maximum Likelihood Estimates of the Initial and Transition Probabilities

The transition probabilities  $P_{ij}(t)$  can be estimated by maximizing (6) with respect to  $P_{ij}(t)$  subject to the condition that  $P_{ij}(t) \geq 0$  and  $\sum_{j=1}^2 P_{ij}(t) = 1$ ;  $i = 1, 2$ . When the  $n_{ij}(t)$  are the actual number of years that state  $i$  occurred at time  $t-1$  and state  $j$  occurred at time  $t$ , then the maximum likelihood estimates of  $P_{ij}(t)$  are  $\hat{P}_{ij}(t)$  where

$$\hat{P}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t-1)} = \frac{n_{ij}(t)}{\sum_{k=1}^2 n_{ik}(t)}$$

Thus the estimates  $\hat{P}_{ij}(t)$  of  $P_{ij}(t)$  are just the relative frequencies of state  $i$  at time  $t-1$  and state  $j$  at time  $t$ . The same holds true to the initial probabilities; i.e., if  $P_i(t)$  is the probability of state  $i$  at time  $t$ , its estimate  $\hat{P}_i(t)$  would be

$$\hat{P}_i(t) = \frac{n_i(t)}{n}$$

where

$$\begin{aligned} n_i(t) &= \text{actual number of years that state } i \text{ occurred} \\ n &= \text{number of years of record} \end{aligned}$$

Specifically, we have the following estimates of the different probabilities:

$$\begin{aligned} \hat{P}(D_t) = \hat{P}_1(t) &= \text{estimate of the probability that the } (t)^{\text{th}} \text{ day is dry} \\ &= \frac{\text{Number of years that } (t)^{\text{th}} \text{ day is dry}}{\text{Number of years of record}} \end{aligned} \quad (7)$$

$$\begin{aligned} \hat{p}(D_t|d_{t-1}) = \hat{P}_{11}(t) &= \text{estimate of the probability that the } (t)^{\text{th}} \\ &\quad \text{day is dry given that the } (t-1)^{\text{th}} \text{ day was} \\ &\quad \text{dry.} \\ &= \frac{\text{Number of years that the } (t)^{\text{th}} \text{ and } (t-1)^{\text{th}} \text{ were dry}}{\text{Number of years that } (t-1)^{\text{th}} \text{ day was dry}} \end{aligned} \quad (8)$$

Estimates of the other probabilities associated with a first order Markov chain can be computed from the above estimates. They are as follows:

$$\begin{aligned} \hat{P}(W_t) &= 1 - \hat{P}(D_t) \\ \hat{P}(W_t/D_{t-1}) &= 1 - \hat{P}(D_t/D_{t-1}) \\ \hat{P}(D_t/W_{t-1}) &= [\hat{P}(D_t) - \hat{P}(D_{t-1})\hat{P}(D_t/D_{t-1})]/(1 - \hat{P}(D_{t-1})) \\ \hat{P}(W_t/W_{t-1}) &= 1 - \hat{P}(D_t/W_{t-1}) \end{aligned}$$



The same procedure holds true with the estimates of the probabilities associated with the second-order Markov chain, i.e.,

$\hat{P}(D_t|D_{t-2} D_{t-1}) = P_{111}(t)$  = estimates of the probabilities that the  $(t)^{th}$  day is dry given that the  $(t-1)^{th}$  and  $(t-2)^{th}$  days were dry.

$$= \frac{\text{number of years that the } (t)^{th}, (t-1)^{th} \text{ and } (t-2)^{th} \text{ days were dry}}{\text{number of years that the } (t-1)^{th} \text{ and } (t-2)^{th} \text{ days were dry.}}$$

## 2.2 Test on the Order of the Markov Chain Model

Anderson and Goodman (1957) proposed a simple test for testing the appropriate order of a Markov chain. It consists of computing a test statistic that is asymptotically distributed as a chi-square variate and it is computed from ordinary contingency tables. In particular, suppose it is desired to compare the goodness of fit between a simple and a second-order Markov chain model. Let  $n_{ijk}(t)$  be the number of observation (years) in state  $i$  at  $t-2$ , in  $j$  at  $t-1$ , and in  $k$  at  $t$ , and let

$$n_{ij}(t-1) = \sum_{k=1}^2 n_{ijk}(t)$$

$P_{ijk}(t)$  = probability of being in state  $k$  at  $t$  given that it is in state  $i$  at  $t-2$  and in  $j$  at  $t-1$ .

Then the maximum likelihood estimate of  $P_{ijk}(t)$  is  $\hat{P}_{ijk}(t)$  where

$$\hat{P}_{ijk}(t) = \frac{n_{ijk}(t)}{n_{ij}(t-1)}$$

The null hypothesis to be tested is:

$$P_{ijk} = P_{2jk} = P_{jk} \text{ for } j, k = 1, 2.$$

The likelihood ratio for testing this hypothesis is

$$\lambda = \sum_{i,j,k=1}^2 (P_{ij} / P_{ijk})^{n_{ijk}}$$

where  $P_{ij}$  is as defined earlier. Under the null hypothesis,  $-2 \log \lambda$  has an asymptotic chi-square distribution with  $2(2 - 1)^2$  degrees of freedom. Note that the likelihood criterion resembles likelihood ratios obtained for problems relating to contingency tables. The similarity with the standard procedures for contingency tables is as follows:

For a given  $j$ , a  $2 \times 2$  table can be used to represent the estimates  $\hat{P}_{ijk}$  for  $i, k = 1, 2$ . The null hypothesis is:

$$P_{ijk} = P_{jk} \text{ for } i = 1, 2$$

and so the chi-square test of homogeneity seems appropriate. To test this hypothesis, calculate

$$\chi_j^2 = \sum \sum n_{ij}^* (\hat{P}_{ijk} - \hat{P}_{jk})^2 / \hat{P}_{jk}$$

where

$$n_{ij}^* = \sum_{k=1}^2 n_{ijk}$$

If the hypothesis is true,  $\chi_j^2$  has the usual limiting distribution with  $(2 - 1)(2 - 1)$  degrees of freedom.

To test the joint hypothesis that  $P_{ijk} = P_{jk}$  for all  $i, j, k = 1, 2$ , compute the test statistic chi-square, where

$$\chi^2 = \sum_{j=1}^2 \chi_j^2$$

which has the usual limiting distribution with  $2(2 - 1)^2$  degrees of freedom.

### 3. RESULTS AND DISCUSSION

#### 3.1 Estimation of Initial and Transition Probabilities:

Data consisting of daily rainfall amounts from 40 weather stations were used to estimate the initial and transition probabilities of the said stations. The author however decided to include only the probability table (Table 5) for the Davao station due to space constraint. Information on the other station can be secured from the Department of Statistics and Statistical Laboratory, College of Arts and Sciences, U.P. at Los Baños.

The probability table consists of five columns representing five different thresholds for a day to be considered dry or wet. Thus a day is considered wet if the amount of rainfall is greater than or equal to .02 inch, .05 inch, .10 inch and .50 inch. This has been done to give more flexibility in problem solving since not all operations are affected to the same degree by the same amount of rainfall. Each of the five sets further consists of four columns corresponding to the probabilities as follows:

- $D$  = probability that the  $(t)^{th}$  day is dry.
- $D/D$  = probability that the  $(t)^{th}$  day is dry given that the  $(t-1)^{st}$  day is dry.
- $D/DD$  = probability that the  $t^{th}$  day is dry given that both the  $(t-1)^{st}$  and  $(t-2)^{nd}$  days are dry.
- $D/WD$  = probability that the  $(t)^{th}$  day is dry given that the  $(t-1)^{st}$  day is wet and  $(t-2)^{nd}$  day is dry.

For example, if a dry day is defined to be one with less than .02 inch of rainfall, in Davao:

1. The probability that March 10 is dry is .63.
2. The probability that March 10 is dry given that March 9 is dry is .65.
3. The probability that March 10 is dry given that March 8 and March 9 are both dry is .67.
4. The probability that March 10 is dry given that March 9 is wet and March 8 is dry is .66.

Notice also that the probabilities range from 1 to 99. In the preliminary run of the data, the probabilities ranged from 0 to 100. This could be due to insufficient length of records making it possible for certain events to have happened constantly or not happen within the span of time that was considered in this paper. To avoid such figures, the author redefined the probabilities as follows: If  $P$  is the computed probability estimate, then the new estimate  $\bar{P}$  is given by

$$\bar{P} = \begin{cases} .01, & \text{if } \hat{P} = 0 \\ .99, & \text{if } \hat{P} = 100 \\ \hat{P}, & \text{Otherwise} \end{cases}$$

This has been done to give more realism when speaking of the probability of occurrence of weather events. Furthermore, .01 and .99 are assumed small and large enough in place of 0 and 1 respectively.

### 3.2 Directions for use of the probability table

If one is only interested in the probability that a particular day will be dry (or wet) simply read the entry in which the day falls. However, many problems involve periods of two or more days. For illustration, assume that one is interested in the rainfall pattern of Davao during April 17-19. Let a dry day be one with less than .50 inch of rain. Then there are eight possible sequences of three dry and/or wet days namely:  $S_1(D, D, D)$ ,  $S_2(D, D, W)$ ,  $S_3(D, W, D)$ ,  $S_4(D, W, W)$ ,  $S_5(W, D, D)$ ,  $S_6(W, D, W)$ ,  $S_7(W, W, D)$ , and  $S_8(W, W, W)$ . The probability for any one of these sequences is computed by taking the product of appropriate initial and transition probability obtained from Table 5. More specifically,

$$\begin{aligned} \hat{P}(S_1) &= \hat{P}(D_{17}) \cdot \hat{P}(D_{18}|D_{17}) \cdot \hat{P}(D_{19}|D_{18}, D_{17}) \\ &= (.89) (.81) (.93) = .670437 \end{aligned}$$

$$\begin{aligned} \hat{P}(S_2) &= \hat{P}(D_{17}) \cdot \hat{P}(D_{18}|D_{17}) \cdot [1 - \hat{P}(D_{19}|D_{18}, D_{17})] \\ &= (.89) (.81) (.07) = .050463 \end{aligned}$$

$$\begin{aligned} \hat{P}(S_3) &= \hat{P}(D_{17}) \cdot \hat{P}(W_{18}|D_{17}) \cdot \hat{P}(D_{19}|W_{18}, D_{17}) \\ &= (.89) (1-.81) (.89) = .150499 \end{aligned}$$

$$\begin{aligned}\hat{P}(S_4) &= \hat{P}(D_{17}) \cdot \hat{P}(W_{18}|D_{17}) \cdot [1 - \hat{P}(D_{19}|W_{18}, D_{17})] \\ &= (.89)(1-.81)(1-.89) = .015219\end{aligned}$$

$$\hat{P}(S_5) = \hat{P}(D_{18}, D_{19}) - \hat{P}(D_{17}, D_{18}, D_{19})$$

$$\hat{P}(S_6) = \hat{P}(D_{18}, W_{19}) - \hat{P}(D_{17}, D_{18}, W_{19})$$

$$\hat{P}(S_7) = \hat{P}(W_{18}, D_{19}) - \hat{P}(D_{17}, W_{18}, D_{19})$$

$$\hat{P}(S_8) = \hat{P}(W_{18}, W_{19}) - \hat{P}(D_{17}, W_{18}, W_{19})$$

where

$$\hat{P}(D_{18}, D_{19}) = \hat{P}(D_{18}) \cdot \hat{P}(D_{19}|D_{18}) = (.81)(.91) = .7371$$

$$\hat{P}(D_{18}, W_{19}) = \hat{P}(D_{18}) \cdot [1 - \hat{P}(D_{19}|D_{18})] = (.81)(1-.91) = .0729$$

$$\hat{P}(W_{18}, D_{19}) = \hat{P}(D_{19}) - \hat{P}(D_{18}, D_{19}) = .91 - .7371 = .1729$$

$$\hat{P}(W_{18}, W_{19}) = \hat{P}(W_{19}) - \hat{P}(D_{18}, W_{19}) = (1-.91) - .0729 = .0171$$

Hence,

$$\hat{P}(S_1) = .670437 \quad \hat{P}(S_2) = .050462 \quad \hat{P}(S_3) = .150499$$

$$\hat{P}(S_4) = .015219 \quad \hat{P}(S_5) = .066663 \quad \hat{P}(S_6) = .022437$$

$$\hat{P}(S_7) = .022401 \quad \hat{P}(S_8) = .001881$$

The above computational technique is based on the assumption that the rainfall pattern of Davao follows a second-order Markov chain model. However, if a simple order Markov chain is sufficient, as will be determined in the next section, then the computation can be simplified. For example,

$$\hat{P}(S_2) = \hat{P}(D_{17}) \cdot \hat{P}(D_{18}/D_{17}) \cdot \hat{P}(W_{19}/D_{18})$$

$$\hat{P}(S_7) = \hat{P}(W_{17}) \cdot \hat{P}(W_{18}/W_{19}) \cdot \hat{P}(D_{19}/W_{18})$$

The other probabilities can be obtained in a similar manner.

The following examples will also suggest the other uses of the probability table.

*Example 1.*

In Davao, upland rice is usually planted between September 10 and September 20. Once the seed is in the ground, the farmer would welcome rainfall. What is the probability of no rains exceeding .20 inch per day occurring in the fourteen day period following planting assuming that seed broadcasting is done on September 12.

*Solution:*

An approximate answer can be obtained using Table 5. The probability sought is that of a sequence of 14 days. Assume also that the crop will be planted on a dry day so that the transition probability  $P(D_t|D_{t-1})$  and  $P(D_t|D_{t-1}, D_{t-2})$  will be used for all 14 days rather than just the last 13 days. Thus, if the seeds are broadcasted on September 12, then the probability of 14 consecutive dry days will be

$$(.76) (.77) (.81) \dots (.69) = .048$$

Hence, on the average, in about 5 years out of 100, 14 days with no rains exceeding .20 inch per day will follow the day of planting. This will also mean that there will be at least one day with rainfall greater than .20 inch in the two weeks following the day of planting in about 95 years out of 100.

*Example 2.*

The probability that can be read and computed from the table can be more significant if such figures can be related with the economic gains or losses of some activities. Such economic gains or losses may not be in terms of pesos but in terms of yield of some crops, attendance figures, or any unit which is sensible for a given problem.

The formula for computing the average of a three-day period may be given as follows:

$$\text{Estimated Value} = [E_1 \cdot \hat{P}(S_1) + E_2 \cdot \hat{P}(S_2) + \dots + E_8 \cdot \hat{P}(S_8)]$$

since there are eight possible sequences of wet and/or dry days on a three-day period, and  $S_i$  is the  $i^{th}$  sequences defined earlier,  $E_i$  is the economic gain associated with the  $i^{th}$  sequence, and  $P(S_i)$  is the probability of occurrence of  $S_i$ .

To illustrate the use of such formula, consider the problem of estimating the average attendance of a three-day baseball tournament in Davao to be held on April 17, 18 and 19. Suppose that a rain in excess of .50 inch on a particular day results in the cancellation of the games for that day. The expected attendance and other calculations are shown below.

<u>Condition</u>	<u>Probabilities</u>	<u>Expected Attendance</u>
0 wet day	$\hat{P}(S_1)$	= .570437 (6000)
1 wet day	$\hat{P}(S_2) + P(S_3)$ + $\hat{P}(S_5)$	= .267625 (4000)
2 wet days	$\hat{P}(S_4) + \hat{P}(S_6)$ + $\hat{P}(S_7)$	= .60057 (2000)
3 wet days	$\hat{P}(S_8)$	= .001881 (0)

The average attendance will be the sum of the expected attendance for the different conditions which turns out to be 5213.

The calculated average attendance may not be exact attendance for the said event. If the figures are realistic, it could either be 6000, 4000, 2000 or 0. However if such event is held year after year with the same expected attendance then the average attendance over a large number of years would be around 5213.

It may be argued that the example oversimplifies the real situation to some extent. A strong rain in the morning and a sudden change of weather in the afternoon may not cancel games scheduled in the afternoon. The attendance figures may also be underestimated or overestimated depending on some factors other than rainfall. Despite these short-comings, the knowledge of rainfall probabilities may help assess the uncertainty of surrounding events thus reducing the chance of planning an event with a sizeable probability of being a "financial disaster" due to whims of nature.

### 3.3 Statistical Dependence – a test on the order of the Markov Chain

A comparison on the goodness of fit between a simple and second order Markov chains was carried out by computing a test statistic that is asymptotically distributed as a chi-square variate. Such test statistic was simply obtained from an ordinary contingency table. Thus, fifty years of record, for example, would provide fifty observations for enumerating the number of times each of the eight sequences. Fifty sample points however is relatively few as some particular sequence may not appear at all. Another fifty observations can be added by considering the sequence  $(X_{t-1}, X_t, X_{t+1})$  and the counts obtained from the sequence is asymptotically independent from those of the preceding sequence (Anderson and Goodman). The addition of more observations may be continued by varying  $t$  say over forty days.

In this case two thousand observations would be available to test the null hypothesis

$$H_0: P(X_t | X_{t-2}, X_{t-1}) = P(X_t | X_{t-1}).$$

against the alternative hypothesis

$$H_a: P(X_t | X_{t-2}, X_{t-1}) \neq P(X_t | X_{t-1})$$

Results of the computation for Tagbilaran under a forty-day period starting with day 111 and ending with day 150 are shown in Table 1. Notice that there are two chi-square values computed; one for sequences in which the middle day ( $t-1$ ) is dry and the other in which the middle day is wet. The two provide asymptotically independent tests of the same null hypothesis,  $H_0$  which could be stated as follows: Given the weather ( $D$  or  $W$ ) on the middle day of a three-day sequence, the weather ( $d$  or  $W$ ) on the third day is independent of the weather ( $D$  or  $W$ ) on the first day. Under a five percent level of significance, the null hypothesis would be rejected for the sequences in which the middle day is dry but not for the sequences where the middle day is wet.



A logical question that is posed at this point is whether season or time of the year has something to do with the dependency of the conditional probabilities. In this aspect, the author selected four different periods, each period consisting of forty consecutive days. The selection of the periods is however based on the type of climate to which a station belongs while the selection of the starting day is random. Thus for type I climate which is characterized by two pronounced seasons, one period is selected during the dry season, another within the wet season, and the other two periods during the transition from wet to dry season and vice versa. For the type II climate, one period is located within the month of November to January as this is the period of maximum rainfall while the other periods are selected randomly during the rest of the year. Under the type III climate, two periods are selected within the months of November to April as these are the relatively dry months although the seasons are not really pronounced. The four periods in the type IV climate which is characterized by an even distribution of rainfall throughout the year are likewise evenly allocated throughout the whole year. The different climates together with their correspondings periods are shown in Table 2.

Table 1. OBSERVED NUMBERS FOR COMPUTING CHI-SQUARE STATISTICS  
(Tagbilaran-threshold of .20 inch)

		DRY <sub>(t-1)</sub>					WET		
<i>X<sub>t-2</sub></i> \ <i>X<sub>t</sub></i>		<i>D</i>	<i>W</i>	<i>Total</i>	<i>X<sub>t-2</sub></i> \ <i>X<sub>t</sub></i>	<i>D</i>	<i>W</i>	<i>Total</i>	
<i>D</i>		979	224	1203	<i>D</i>	197	105	302	
<i>W</i>		224	77	301	<i>W</i>	105	49	154	
<b>Total</b>		1203	301	1504	<b>Total</b>	302	154	456	

$$\chi_D^2 = 6.860$$

$$\chi_W^2 = 0.276$$

Table 2. SELECTED FORTY-DAY PERIODS FOR CHI-SQUARE ANALYSIS

(Entries are days of the year beginning with March 1 = 1)

<i>Selected Forty-Day Period</i>				
Climate	1	2	3	4
Type I	41-80	121-160	231-270	301-340
Type II	26-65	111-150	261-300	326-365
Type III	31-70	131-170	251-290	321-360
Type IV	21-60	111-150	201-240	301-340

Results of the chi-square analysis on the depending of conditional probabilities of rainfall for selected weather stations are shown in Table 3. The figures presented however are joint chi-square values, meaning, the sums of pairs of chi-square values ( $\chi_D^2$   $\chi_W^2$ ) like the ones appearing in Table 1. A significant figure in Table 3 implies that either or both  $\chi_D^2$  and  $\chi_W^2$  are significant which would further imply a rejection of the null hypothesis  $H_0$ . In almost all cases,  $\chi_D^2$  and  $\chi_W^2$  are both significant thus resulting in large magnitude of the joint chi-square values. This would therefore mean that the order of the Markov chain is at least two. On the other hand, accepting the hypothesis on the basis of a small chi-square value does not prove that we are dealing with a first order chain but suggests that observed departures from a first-order Markov chain model may simply be due to random errors.

Further scrutiny of the analysis showed that the chi-square values of almost all the periods in all thresholds for stations belonging to the type I climate are significant at the five percent level. The stations belonging to the type II climate show a different trend in the sense that the early periods seem to give more significant results than the later ones as exemplified by Infanta and Virac. This is however the opposite with stations belonging to the type IV climate i.e., the later periods are showing more significant results than the early ones as in the case of Dipolog and Jolo. Results for

the stations of the type III climate do not give a clear picture as to the trend of significant values with respect to the periods where the null hypothesis may be rejected or accepted.

A summary of the results of the chi-square analysis concerning significant values is shown in Table 4.1 and 4.2. The result shows higher percentage of significant chi-square values for lower thresholds and early periods while lower percentage appears in the later periods and higher thresholds. Under .02 inch threshold and period I, 85 percent (maximum) of the stations involved in the study showed significant results when using .05 level of significance. This overwhelmingly suggests the use of at least a second order chain when describing sequences of wet and dry days during the early part of the year, i.e., March to April and when a wet day is defined in terms of small thresholds. The exact opposite, however, is seen under threshold .50 inch and period III; only 20 percent (minimum) of the forty stations registered significant results. In this case, one can safely use a first-order chain for higher thresholds and during the later periods, i.e., November to January.

Tables 4.1 and 4.2 also show a decreasing trend in the percentage values as one moves from period I to period IV. Regardless of threshold, 73.75 percent of all the chi-square values under period I using .05 level of significance and 80 percent when using .10 level of significance while it is only 50 percent ( $\alpha = .05$ ) and 59 percent ( $\alpha = .10$ ) for period III. Similar trend is being exhibited with respect to threshold; regardless of period, there exists an inverse relationship between threshold and the percentage of significant chi-square values. Using .05 level of significance around 71 percent of all the chi-square values under threshold .02 inch show significant results while only 36 percent is registered under threshold .50 inch.

Table 3. COMPUTED CHI-SQUARE VALUES FOR THE TEST ON THE ORDER OF THE MARKOV CHAIN

<i>Wet Day Defined to be:</i>						
<i>Stations</i>	<i>Period</i>	$\geq .02$	$\geq .05$	$\geq .10$	$\geq .20$	$\geq .50$
Baguio	I	29.045	29.877	19.233	15.260	12.409
	II	17.709	12.796	21.054	24.425	30.389
	III	10.092	12.112	9.974	3.407	0.699
	IV	2.272	4.134	10.894	18.327	0.088
Cabanatuan	I	28.183	30.408	34.205	28.739	24.703
	II	39.115	24.274	9.720	7.133	0.331
	III	6.771	6.217	4.949	7.151	5.945
	IV	6.572	0.496	0.243	2.421	1.318
Cagayan de Oro	I	10.054	3.444	6.032	13.806	14.246
	II	1.450	1.937	2.425	3.465	0.180
	III	17.014	12.796	8.572	7.940	6.851
	IV	56.592	39.379	32.905	11.137	12.295
Cebu	I	25.390	19.031	17.541	7.378	3.810
	II	17.538	10.663	11.115	2.958	1.808
	III	6.590	8.475	1.424	4.694	0.572
	IV	9.634	13.698	12.474	6.278	0.469
Davao	I	8.769	10.309	12.964	3.876	22.265
	II	15.472	7.610	3.800	2.056	2.252
	III	6.955	10.153	12.758	3.946	12.905
	IV	13.676	10.612	9.452	6.917	1.075
Dipolog	I	7.545	13.176	15.467	5.484	5.746
	II	0.720	0.740	11.872	13.595	10.613
	III	14.335	9.084	9.010	11.207	5.908
	IV	81.226	77.511	36.681	25.835	5.473
Infanta	I	13.625	17.713	12.632	8.822	8.681
	II	4.734	2.016	8.814	8.123	0.523
	III	1.178	1.416	0.054	0.114	0.428
	IV	0.004	0.094	0.251	0.017	5.143

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*Wet Day Defined to be:*

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<i>Stations</i>	<i>Period</i>	$\geq .02$	$\geq .05$	$\geq .10$	$\geq .20$	$\geq .50$
Manila	I	28.922	41.409	27.851	29.374	7.808
	II	18.141	14.897	40.847	36.306	36.224
	III	14.192	12.959	6.950	3.556	1.170
	IV	3.996	1.444	2.351	1.798	0.048
Tagbilaran	I	28.726	23.760	25.842	30.298	14.922
	II	8.107	6.575	14.521	7.136	0.840
	III	0.933	2.266	1.010	3.232	0.182
	IV	9.532	9.560	14.163	14.858	12.694
Zamboanga	I	3.708	8.509	5.601	6.146	1.254
	II	7.831	6.407	0.077	0.826	0.355
	III	2.880	2.136	6.162	11.713	1.425
	IV	10.572	9.638	6.799	2.157	0.007

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Table 4.1 PERCENTAGE OF SIGNIFICANT CHI-SQUARE VALUES FOR  
THE TEST ON THE ORDER OF THE MARKOV CHAIN USING  
.05 LEVEL OF SIGNIFICANCE

<i>PERIOD</i>					
<i>Threshold</i>	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>	<i>Mean</i>
.02 in	85.00	67.50	67.50	65.00	71.25
.05 in	82.50	65.00	62.50	60.00	67.50
.10 in	82.50	55.00	57.50	57.50	63.13
.20 in	65.00	52.50	45.00	47.50	52.52
.50 in	45.00	42.50	20.00	37.50	36.25
Mean	73.75	56.50	50.50	53.50	58.13

Table 4.2 PERCENTAGE OF SIGNIFICANT CHI-SQUARE VALUES FOR  
THE TEST ON THE ORDER OF THE MARKOV CHAIN USING  
.10 LEVEL OF SIGNIFICANCE

<i>PERIOD</i>					
<i>Threshold</i>	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>	<i>Mean</i>
.02 in	87.50	75.00	70.00	72.50	76.25
.05 in	90.00	70.00	67.50	62.50	72.50
.10 in	87.50	57.50	67.50	62.50	68.75
.20 in	85.00	57.50	57.50	60.00	65.00
.50 in	50.00	45.00	32.50	47.50	43.75
Mean	80.00	61.00	59.00	61.00	65.25

*SUMMARY AND CONCLUSION*

The rainfall pattern of the Philippines was studied based on observations gathered from forty major stations throughout the country. The probability of occurrence for a given consecutive sequence of wet and dry days beginning with any day of the year was computed for each station. The specific probabilities considered based on relative frequency are  $P(D_t)$ ,  $P(D_t|D_{t-1})$ ,  $P(D_t|D_{t-1}, D_{t-2})$  and  $P(D_t|W_{t-1}, D_{t-2})$ . The other probabilities such as  $P(W_t)$ ,  $P(W_t|W_{t-1}, W_{t-2})$  and the rest of the transition probabilities of the second-order were dropped since they are functionally related to the former set of probabilities.

A test on the order of the Markov Chain that describes Philippine rainfall pattern shows that one cannot make any generalization regarding the specific order that must be used since only 58 percent ( $\alpha = .05$ ) and 65 percent ( $\alpha = .10$ ) of the possible combinations of thresholds and periods in all forty stations indicated the use of at least a second order chain. However, when a wet day is defined in terms of low thresholds, a sequence of wet and dry days during the early part of the year is best described by at least a second-order chain while a first order chain is sufficient to described sequences during the later part of the year and a wet day is defined in terms of higher thresholds. Thus, the order of the chain is dependent on the time of the year as well as the threshold for a wet day. In view of the above results, determining the distribution function of the largest daily amount as well as the total amount of rainfall in an  $n$ -day period based upon a stochastic model of  $n$ -day rainfall, would be a fitting extension of this study. Furthermore, a possible improvement on the estimates of the different initial and transition probabilities can be done by fitting partial sums of Fourier series to them. This can lead to a considerable reduction in the variance of the estimates.

TABLE 5. INITIAL AND TRANSITION PROBABILITIES – DAVAO

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
MAR 1	66	68	72	79	66	65	72	87	69	67	72	99	76	77	77	99	89	90	91	99
2	71	77	75	71	71	77	75	68	73	76	76	64	79	82	95	69	91	94	95	66
3	79	90	90	33	79	90	90	33	83	90	90	59	38	93	94	75	93	88	98	66
4	63	70	76	1	66	72	79	25	69	75	79	25	73	77	79	33	86	89	88	99
5	79	78	79	85	79	79	79	84	81	83	84	75	36	90	90	66	96	96	95	99
6	71	70	79	62	75	75	78	62	78	77	82	71	81	78	84	99	89	89	91	99
7	64	65	70	71	69	68	72	83	76	78	81	81	88	91	90	72	93	96	96	66
8	76	76	78	79	79	78	80	85	81	82	81	79	86	86	84	75	93	92	92	99
9	71	73	63	99	76	81	75	77	78	83	91	75	81	84	84	71	89	89	88	99
10	63	65	67	66	63	65	69	66	73	74	75	75	76	77	77	75	84	83	83	99
11	68	73	67	59	69	76	73	62	75	81	79	66	76	82	81	63	88	88	88	88
12	75	80	78	59	78	83	82	55	79	79	90	75	86	86	84	87	93	94	93	83
13	68	68	69	75	69	70	68	85	78	79	80	77	84	84	82	83	86	85	83	99
14	69	70	70	71	73	73	75	71	73	74	73	69	75	76	77	62	88	92	91	62
15	71	76	72	50	73	77	74	54	79	84	79	66	83	86	84	66	93	92	91	99
16	81	90	87	59	83	88	85	69	86	89	86	71	89	91	89	66	91	92	91	75
17	75	77	84	75	75	75	84	59	83	84	86	79	83	85	86	75	88	89	88	75
18	64	68	71	63	66	71	73	66	75	71	75	99	79	77	78	99	83	81	81	99
19	68	71	74	57	68	69	71	61	71	75	72	57	78	81	79	63	83	83	81	79
20	73	78	78	63	75	80	82	66	78	83	85	63	81	87	84	66	93	91	90	99
21	63	61	62	66	66	64	63	62	73	72	72	71	79	79	78	66	89	89	89	99
22	76	81	77	64	79	84	82	64	89	95	94	76	93	99	99	69	94	94	93	99
23	78	78	83	71	78	79	82	66	81	83	83	50	91	92	91	1	94	94	94	99
24	79	85	88	59	79	85	86	59	88	93	95	66	93	94	96	99	94	94	98	99
25	76	83	82	57	76	83	82	57	79	84	82	33	81	82	80	66	93	92	92	99
26	75	76	75	62	78	78	77	62	79	83	82	62	79	83	82	69	89	92	92	50
27	81	79	85	81	80	81	86	79	83	83	89	75	89	91	95	75	94	94	94	99
28	71	71	75	66	75	73	75	77	75	71	75	87	81	81	86	75	86	87	90	66
29	64	72	65	50	69	77	72	53	73	82	77	50	78	83	81	59	86	94	93	42
30	63	64	64	66	64	66	65	69	66	70	67	62	71	76	75	50	84	86	85	99
31	69	76	87	64	75	82	89	71	75	82	90	69	78	83	86	63	86	88	88	71



Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
APR 1	78	85	82	55	79	86	84	57	83	91	90	57	84	89	88	71	94	96	95	83
2	71	76	77	66	71	77	79	66	76	81	85	75	78	84	85	59	99	92	93	1
3	81	81	77	36	73	83	81	36	79	82	82	55	84	87	86	62	93	92	92	99
4	75	79	79	75	75	77	77	85	76	79	78	75	79	78	80	99	89	89	89	99
5	71	73	76	44	71	73	76	50	75	76	76	59	76	77	77	72	89	88	87	99
6	69	79	78	50	73	81	81	58	78	84	82	72	81	86	86	81	91	92	93	83
7	86	90	94	99	86	90	94	99	88	89	97	99	91	93	99	83	93	92	95	99
8	68	75	71	25	69	76	72	25	78	81	78	59	59	79	81	80	66	89	92	50
9	73	75	74	69	75	76	75	75	83	82	83	79	86	85	86	89	89	88	90	99
10	78	79	83	79	78	79	84	69	81	81	84	75	84	84	82	85	88	88	89	83
11	69	76	85	44	69	76	83	44	73	79	82	33	81	86	88	50	89	92	93	66
12	68	71	72	63	68	71	72	63	69	75	76	59	75	81	84	42	88	90	91	50
13	79	80	76	83	79	80	76	83	81	80	78	90	84	86	87	88	93	92	93	99
14	66	68	63	62	68	70	63	62	69	71	64	62	75	74	71	83	86	85	87	99
15	63	75	72	46	64	78	76	42	75	85	82	57	84	91	89	69	89	92	91	75
16	73	81	86	50	73	79	81	55	78	82	83	33	83	84	82	50	84	87	87	50
17	76	81	83	85	76	81	83	87	78	80	89	87	83	85	83	87	89	90	89	99
18	58	60	66	62	61	63	69	62	63	63	71	66	66	65	69	71	81	81	84	79
19	63	71	85	55	64	70	82	58	71	76	83	58	83	84	87	76	91	91	93	89
20	56	63	63	39	61	64	65	54	71	76	72	77	75	75	79	83	81	79	84	99
21	59	64	70	64	63	70	75	64	64	65	76	61	71	71	76	75	79	83	88	63
22	64	75	63	66	69	78	73	81	73	76	75	73	79	83	81	76	93	95	95	99
23	56	58	55	55	63	61	66	62	69	68	69	77	71	70	75	71	84	83	89	99
24	58	61	60	68	58	60	61	68	59	61	59	71	66	72	70	64	78	80	82	66
25	75	79	90	53	75	79	91	59	81	83	88	75	83	84	83	75	96	95	97	99
26	58	66	67	14	58	66	67	14	64	73	76	1	68	73	16	88	89	88	88	50
27	61	65	66	59	63	68	66	59	71	74	75	76	76	78	81	76	81	79	80	99
28	63	67	78	50	63	68	79	45	66	72	75	50	68	71	68	55	76	77	78	72
29	63	73	83	50	64	73	84	58	73	79	83	58	75	80	81	53	84	86	89	81
30	61	60	57	69	61	61	57	61	66	70	65	62	69	73	69	62	84	84	82	83

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
MAY 1	58	56	56	59	59	56	54	66	59	54	51	69	69	69	66	66	75	76	76	62
2	56	59	61	62	56	58	61	62	59	61	63	66	64	64	68	76	81	82	82	83
3	63	64	80	64	64	64	80	66	71	69	86	78	79	76	96	93	81	81	89	99
4	61	63	54	41	61	64	54	41	71	72	67	63	73	72	69	77	86	87	87	88
5	69	72	75	64	71	72	75	71	76	76	77	75	81	79	82	884	89	90	90	83
6	51	52	55	29	53	51	55	39	59	54	60	69	61	59	62	66	79	77	76	99
7	56	64	68	50	61	65	68	61	61	63	67	52	69	75	72	54	56	91	90	66
8	51	55	59	45	54	59	57	54	59	64	65	61	61	66	67	66	75	75	81	75
9	56	58	63	73	58	57	63	73	69	69	70	84	75	75	75	92	88	91	92	84
10	50	58	77	46	50	59	78	42	56	66	83	45	61	68	78	55	75	79	75	50
11	48	53	44	35	56	59	52	50	63	67	67	50	66	64	67	64	76	77	80	81
12	58	58	62	42	61	61	66	50	69	73	73	54	78	82	79	69	89	93	94	79
13	56	57	58	58	56	59	61	46	59	61	64	39	68	72	78	28	81	81	83	99
14	43	35	34	46	46	38	36	46	69	71	64	62	75	74	71	83	86	85	87	99
15	54	53	33	59	58	53	38	66	58	55	47	68	71	68	61	79	81	82	79	86
16	56	69	71	58	56	65	73	61	66	77	78	59	68	76	79	54	76	81	85	57
17	51	52	56	29	51	52	56	41	58	59	66	50	68	70	81	59	83	86	94	66
18	48	54	44	37	53	61	55	43	54	65	62	43	61	65	62	58	79	83	82	83
19	50	51	41	57	56	56	52	75	66	66	65	83	76	78	74	92	89	91	90	87
20	48	63	73	35	50	64	77	35	50	57	68	36	59	63	72	50	75	74	77	75
21	64	72	68	72	66	76	77	75	69	79	78	64	71	83	82	64	81	84	84	78
22	51	51	66	87	56	59	73	85	59	59	75	83	63	60	69	66	75	71	73	99
23	59	64	69	68	61	64	66	68	68	75	79	70	78	81	88	82	81	82	85	78
24	64	72	75	63	64	70	72	58	76	78	81	88	84	82	80	99	89	87	86	99
25	59	64	61	50	61	66	65	54	64	67	68	44	66	72	71	37	76	79	79	50
26	61	63	63	57	64	67	65	61	73	74	74	73	83	82	83	78	91	91	93	90
27	54	56	60	53	59	64	67	58	64	65	62	69	69	67	66	85	79	78	83	99
28	53	57	52	50	53	55	47	50	61	58	55	73	66	64	64	75	76	70	69	99
29	58	68	63	57	58	68	59	62	64	72	69	62	75	77	77	79	83	84	91	78
30	61	65	77	59	63	65	77	59	66	71	77	59	73	79	87	66	83	87	87	71
31	50	54	60	50	51	55	65	50	56	54	64	63	63	58	58	77	76	75	75	83

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
JUN 1	43	59	64	29	44	61	66	29	54	58	63	55	68	73	76	66	83	84	81	83
2	44	46	50	58	46	44	52	58	54	54	59	57	68	60	60	89	78	75	79	99
3	59	74	99	50	59	71	99	46	63	69	77	53	75	78	75	68	83	89	89	58
4	46	44	25	85	48	47	25	87	58	60	43	59	69	75	68	44	81	83	83	79
5	54	64	68	34	56	65	70	36	68	68	65	66	78	78	76	63	83	83	83	75
6	53	51	61	69	54	50	63	69	64	60	79	72	69	65	75	88	78	77	80	75
7	58	62	70	62	58	63	70	58	59	64	71	56	64	69	77	56	78	80	87	63
8	63	62	69	75	63	62	71	75	69	69	75	78	73	71	75	84	81	80	81	99
9	41	42	50	38	43	44	50	38	50	50	51	54	66	63	64	81	81	77	81	99
10	54	59	62	59	58	61	64	66	63	63	66	71	69	69	75	75	84	85	86	81
11	56	63	66	59	61	65	75	69	71	68	84	90	79	76	82	91	88	86	90	99
12	58	58	57	75	59	59	60	66	68	67	69	83	71	68	68	79	83	84	81	71
13	64	62	69	85	66	61	68	86	68	68	72	85	71	69	72	79	88	91	93	75
14	51	48	50	69	53	52	54	71	68	73	71	84	76	74	73	92	88	88	86	75
15	66	67	84	69	75	78	85	73	76	76	79	81	88	91	93	90	93	96	97	83
16	54	59	66	50	58	62	67	57	63	67	62	50	71	71	66	50	83	83	84	50
17	43	60	62	25	44	57	53	29	56	57	54	53	59	53	52	79	81	79	80	88
18	54	53	59	46	56	59	64	46	61	61	72	50	64	69	78	54	84	81	84	99
19	50	54	42	50	54	58	50	54	63	67	66	53	73	76	75	63	79	80	79	77
20	46	53	72	19	48	57	69	21	54	57	63	33	63	68	69	44	79	87	85	50
21	46	53	56	35	50	51	52	42	58	60	59	50	71	71	69	71	88	89	90	83
22	58	64	79	69	63	66	79	78	64	65	79	84	71	69	74	81	94	86	83	79
23	51	54	61	39	54	55	54	39	59	61	60	50	66	65	63	69	78	84	84	42
24	53	58	68	56	56	57	66	70	61	61	70	66	71	75	78	66	84	85	88	75
25	54	56	61	61	58	55	63	64	73	75	68	71	76	79	76	79	83	34	87	85
26	46	51	50	57	48	48	47	59	59	54	60	88	64	60	64	88	83	79	79	99
27	51	64	76	43	51	65	76	44	53	69	70	59	71	74	78	66	81	85	89	59
28	58	74	77	59	59	74	78	59	66	71	79	72	78	79	86	79	86	87	90	85
29	51	57	56	50	53	58	56	62	71	72	74	54	73	76	79	55	76	78	79	66
30	53	54	69	59	54	56	71	59	59	60	65	72	66	68	69	72	88	91	82	81

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
JUL 1	50	53	52	53	56	57	50	50	68	72	76	64	69	69	78	78	79	83	83	75
2	53	59	58	46	58	64	57	50	58	63	57	39	69	80	75	50	78	87	88	22
3	63	53	61	75	64	54	59	83	71	63	64	92	75	78	76	87	96	87	90	89
4	56	57	47	58	59	58	52	62	68	72	60	61	78	82	77	63	93	94	95	83
5	48	64	54	37	50	63	56	87	63	65	58	58	73	74	67	75	79	80	79	99
6	61	68	72	50	63	69	78	46	71	76	77	50	79	88	85	41	86	87	86	81
7	58	72	75	55	58	73	76	44	64	74	72	44	68	72	69	59	89	92	92	83
8	61	68	66	59	64	68	67	69	69	74	71	81	76	80	82	84	84	88	91	50
9	66	62	62	81	66	61	62	81	73	69	75	79	78	71	69	99	88	86	85	99
10	56	59	52	35	58	62	54	83	58	61	88	88	69	72	63	61	75	75	72	71
11	64	70	75	56	64	71	75	53	69	74	81	58	76	80	85	61	84	84	82	84
12	59	69	83	29	61	71	83	39	68	76	84	55	73	84	85	37	81	80	92	28
13	69	83	85	58	71	83	85	85	76	85	90	59	78	88	94	42	86	93	97	59
14	58	61	69	16	64	69	70	16	73	76	77	66	76	78	79	59	89	92	91	99
15	69	68	69	68	69	71	69	61	76	75	74	72	81	78	75	89	86	85	85	99
16	69	76	87	54	75	78	85	72	76	80	84	54	83	87	88	59	89	94	93	62
17	63	66	68	50	63	68	69	44	68	71	70	66	79	81	83	66	91	92	91	66
18	66	60	60	78	69	65	61	78	75	75	72	76	78	79	78	66	86	87	85	75
19	61	75	86	39	63	71	79	53	68	73	80	59	73	76	81	59	75	78	79	42
20	56	67	69	19	58	68	69	25	63	70	66	33	68	75	69	45	84	84	82	81
21	64	82	83	50	66	82	84	50	69	81	82	58	75	82	81	63	86	86	89	85
22	58	66	60	16	59	67	58	16	64	73	67	42	71	75	73	57	78	78	77	71
23	58	62	69	38	59	66	74	30	63	69	77	36	71	76	79	54	84	91	90	63
24	66	74	77	53	71	72	70	75	78	78	77	91	79	79	78	99	86	88	93	99
25	68	75	73	55	68	74	73	59	75	80	79	62	76	83	79	55	88	90	88	66
26	64	75	76	59	66	78	78	54	68	75	73	66	75	80	77	87	81	84	87	79
27	66	76	80	50	66	75	78	55	71	75	76	81	78	84	83	77	84	87	88	75
28	64	77	79	55	68	79	79	69	76	76	77	89	94	85	81	85	94	96	95	83
29	69	79	80	66	69	80	81	62	76	84	81	59	78	82	84	57	89	92	93	50
30	61	64	64	75	66	69	69	75	71	76	76	71	78	82	83	77	89	88	90	99
31	79n	89	92	73	81	84	89	84	81	86	88	72	83	85	87	87	86	87	87	83

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
AUG 1	61	64	75	50	63	65	73	50	63	65	70	50	75	75	82	57	86	86	89	85
2	66	70	67	64	66	68	65	64	75	76	75	70	76	77	76	75	81	82	82	85
3	71	72	76	63	71	72	76	66	76	73	75	88	76	73	74	89	86	85	86	88
4	58	67	65	36	58	67	65	36	63	69	66	50	73	75	76	65	89	90	92	85
4	58	67	65	36	58	67	65	36	63	69	66	50	73	75	76	65	89	90	92	85
5	56	57	58	57	59	62	65	57	64	65	71	57	79	79	77	99	86	89	87	79
6	63	64	79	59	66	69	77	61	76	76	76	66	79	83	77	66	88	90	89	66
7	56	60	63	58	61	64	67	63	75	76	76	66	79	81	84	62	91	90	91	99
8	63	61	52	53	63	62	53	57	66	68	65	63	71	72	71	66	81	81	81	79
9	63	65	76	53	66	68	78	57	71	69	74	71	78	76	79	76	79	81	70	69
10	66	76	75	53	66	77	76	50	73	81	82	66	81	85	84	79	89	93	92	88
11	63	69	82	33	64	72	77	33	73	77	77	37	79	79	79	71	88	87	86	99
12	78	81	82	75	81	85	86	72	84	88	91	69	89	91	94	79	93	92	93	99
13	68	72	74	85	71	75	72	83	78	80	82	79	81	81	79	75	86	85	85	99
14	69	73	79	61	71	76	81	58	75	76	80	69	78	79	79	69	83	82	83	87
15	59	64	63	27	59	62	60	29	69	73	72	54	73	76	74	59	88	87	86	88
16	61	55	48	59	61	55	48	62	68	66	63	66	79	79	80	81	86	86	88	83
17	61	72	84	50	63	72	84	56	66	73	82	64	71	72	82	66	81	84	86	57
18	69	75	81	69	71	76	81	60	75	77	79	72	79	83	82	76	86	89	88	87
19	59	64	60	55	63	65	62	55	68	68	67	66	69	70	69	71	81	82	86	79
20	71	83	85	46	73	81	85	53	79	85	83	57	84	90	88	64	93	91	90	99
21	63	72	73	50	64	70	74	57	68	68	71	83	71	72	76	75	79	78	79	99
22	63	65	67	50	64	66	67	53	73	73	72	73	79	79	81	78	88	87	86	91
23	61	63	63	61	63	61	61	61	66	61	63	72	71	70	67	66	81	83	78	66
24	66	78	75	64	66	76	75	66	78	84	85	70	79	81	79	78	89	91	90	77
25	73	72	75	75	78	79	79	77	79	80	76	83	84	85	82	87	80	85	82	89
26	56	61	68	54	58	61	68	62	68	68	71	77	75	76	78	85	84	86	84	75
27	54	64	66	41	56	62	65	44	66	70	72	50	71	79	76	50	81	86	88	57
28	71	81	86	41	71	79	86	46	79	84	82	58	83	88	88	55	88	93	93	42
29	69	74	81	50	73	74	81	57	78	81	85	60	84	85	86	79	86	88	91	66
30	64	71	75	36	68	75	75	36	73	76	76	55	79	82	81	71	86	88	89	83
31	69	82	76	41	69	80	75	45	79	86	83	54	81	87	85	55	86	88	86	83

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
SEPT 1	53	57	59	71	54	59	57	62	58	60	60	66	66	69	73	66	73	73	76	66
2	58	75	70	38	59	72	67	41	66	74	72	63	71	79	79	66	84	90	92	71
3	66	68	66	62	68	72	70	55	69	72	73	66	73	74	68	75	84	86	84	75
4	58	62	70	36	58	63	73	29	69	76	82	36	76	81	81	45	84	90	90	42
5	54	74	71	33	58	77	76	39	63	69	71	50	69	69	69	87	81	82	80	99
6	61	72	73	77	63	71	74	87	73	84	86	76	78	88	90	57	88	91	92	77
7	51	67	66	55	53	68	71	50	56	70	75	16	66	76	78	19	76	79	79	50
8	56	70	71	25	58	68	69	25	64	73	74	46	66	75	75	54	83	84	85	81
9	54	61	72	33	58	62	72	50	63	64	67	77	71	75	76	79	78	77	79	71
10	50	66	71	30	54	71	77	30	59	76	79	28	66	76	79	29	79	80	84	72
11	59	66	72	54	64	66	71	69	69	72	75	77	78	79	78	69	84	85	81	77
12	68	72	79	59	68	71	81	63	73	76	80	59	76	78	87	62	89	88	92	99
13	71	65	65	79	73	68	67	72	78	72	68	89	79	76	72	89	88	88	88	83
14	66	72	70	50	69	75	75	53	71	74	75	58	75	77	77	63	81	81	81	83
15	59	64	64	41	63	69	69	36	66	72	74	50	73	79	81	54	83	91	93	39
16	61	75	76	50	64	78	75	53	76	89	87	66	81	90	88	55	86	91	93	50
17	63	62	70	77	66	69	76	75	73	78	83	99	81	83	84	99	84	88	89	75
18	58	71	82	57	59	72	77	58	66	72	77	69	76	75	78	75	88	88	89	83
19	58	62	70	45	61	63	72	45	66	64	68	75	73	73	75	75	81	83	84	83
20	64	74	72	61	66	72	73	61	66	89	89	57	75	77	73	66	84	87	86	77
21	66	71	73	77	69	75	77	79	75	79	82	75	75	82	85	59	84	88	93	83
22	75	71	85	81	76	78	86	79	83	84	96	87	83	84	89	87	88	88	93	83
23	63	68	74	55	63	67	72	55	69	69	73	57	78	79	84	57	89	92	91	66
24	68	76	77	50	71	81	80	53	73	80	82	53	76	85	84	39	81	85	85	25
25	69	75	82	55	69	74	80	57	78	79	79	75	84	86	87	71	93	93	93	87
26	41	40	45	29	44	40	43	54	53	53	57	55	61	62	69	66	73	73	73	99
27	58	75	76	43	58	74	78	43	59	68	71	50	64	70	75	52	81	88	87	59
28	58	57	57	13	61	62	69	28	63	63	63	50	68	66	65	72	78	77	76	79
29	64	79	79	33	66	78	77	38	73	84	82	46	73	84	83	42	83	87	86	63
30	54	53	60	57	56	54	62	75	58	59	65	86	69	68	74	83	84	85	85	99

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
OCT 1	59	60	61	61	59	58	59	61	71	77	76	66	76	83	86	57	88	92	95	57
2	66	69	69	61	66	69	69	66	73	74	70	87	76	78	77	71	81	84	87	75
3	53	52	43	63	54	54	43	63	63	68	62	45	71	73	69	69	81	85	86	75
4	48	59	66	36	51	63	72	38	61	73	76	50	64	79	76	41	76	85	83	57
5	64	65	57	53	68	70	66	58	78	83	78	69	81	89	88	66	91	97	97	71
6	53	56	57	59	54	58	59	66	63	65	70	83	73	79	82	75	81	83	84	99
7	66	71	68	58	68	69	66	64	78	81	80	68	84	86	84	79	91	91	91	88
8	73	82	91	66	76	80	91	69	79	82	92	71	83	86	89	83	88	92	93	50
9	61	70	72	28	61	71	72	25	73	79	82	62	79	83	81	71	86	85	88	75
10	56	64	64	38	56	64	66	38	66	72	73	50	68	75	76	50	75	78	80	50
11	68	82	79	61	69	82	79	69	73	82	81	66	78	87	86	66	84	88	90	81
12	50	53	50	66	50	52	50	66	54	56	54	57	59	59	58	79	71	70	69	99
13	58	73	68	47	63	73	68	59	66	78	79	57	69	83	82	52	86	93	91	66
14	59	62	59	37	61	63	59	50	68	72	76	42	69	76	76	33	78	78	79	99
15	61	58	50	69	64	64	58	64	69	73	68	63	79	80	81	69	89	91	92	81
16	54	56	76	53	54	58	79	46	64	71	76	36	69	79	79	25	83	87	88	50
17	59	63	66	62	59	63	65	62	64	69	76	58	71	73	76	79	84	91	93	42
18	61	66	71	41	64	69	76	41	75	82	85	50	78	86	87	54	89	92	91	99
19	59	62	58	50	61	61	59	54	63	62	62	57	68	65	62	66	81	79	76	99
20	39	50	47	28	43	54	54	33	53	60	57	41	63	63	61	62	73	69	69	90
21	51	58	55	61	51	61	59	52	54	53	56	66	66	65	69	66	84	81	85	93
22	59	58	71	59	59	58	68	59	61	60	70	53	68	64	63	76	76	76	75	87
23	51	58	77	38	53	61	83	38	64	64	84	69	71	73	76	71	84	84	89	99
24	51	61	57	39	53	65	63	35	59	69	70	46	63	72	73	45	76	76	76	71
25	58	54	68	83	59	59	71	81	69	69	70	91	79	78	83	99	88	86	92	91
26	64	62	52	64	68	66	57	69	71	71	67	63	76	79	76	62	84	84	79	83
27	66	64	72	69	66	63	70	75	71	67	69	75	71	69	68	69	79	78	77	87
28	63	64	63	64	64	67	65	59	66	67	65	64	75	79	81	64	86	89	92	81
29	54	57	53	50	58	56	51	53	68	64	65	71	75	71	73	88	84	84	90	99
30	61	75	77	43	63	74	77	47	68	70	73	57	71	75	75	61	81	80	79	87
31	71	72	83	37	73	76	84	44	76	78	82	66	79	75	79	90	88	85	90	99

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
NOV 1	63	67	74	59	64	68	72	66	78	82	87	66	84	85	90	81	89	90	90	85
2	58	65	62	42	61	69	66	42	69	78	76	25	71	74	75	42	81	83	81	59
3	46	54	67	46	51	56	66	58	54	57	56	69	61	65	63	61	75	79	79	55
4	56	57	57	56	58	54	57	56	71	69	70	72	78	75	75	79	83	79	76	99
5	59	64	68	58	64	68	70	64	75	76	78	79	78	76	82	77	84	81	86	99
6	54	69	72	50	56	69	75	45	59	66	66	59	64	70	72	54	75	76	75	66
7	56	60	59	81	56	58	55	83	69	69	73	66	73	76	75	64	84	84	83	83
8	63	61	59	76	63	61	59	71	68	66	63	81	69	68	63	77	81	78	76	99
9	63	71	71	38	63	71	71	38	68	75	75	42	69	76	76	50	79	79	79	81
10	61	63	70	45	61	63	70	45	63	68	74	39	78	85	84	50	89	89	87	89
11	56	59	50	42	56	59	51	42	66	65	57	61	68	65	61	66	79	79	66	79
12	69	70	81	86	69	70	81	86	73	72	87	84	76	78	83	75	83	83	86	81
13	71	78	79	69	73	78	79	79	76	77	79	81	83	86	87	77	88	89	89	87
14	68	76	75	55	71	77	75	55	73	76	76	59	78	81	82	50	89	88	88	99
15	66	65	69	69	69	69	67	69	73	70	68	81	76	72	70	88	86	85	85	99
16	75	77	77	71	75	76	76	69	79	84	80	61	84	86	82	76	93	92	91	99
17	71	73	77	77	71	73	78	79	81	85	86	71	83	88	89	50	91	92	91	75
18	73	72	69	93	76	76	72	83	81	81	80	85	83	83	82	83	94	94	94	99
19	69	68	77	83	71	69	75	89	78	77	79	88	81	79	80	99	86	85	88	99
20	64	69	66	57	64	69	68	57	79	87	86	54	81	85	84	59	88	90	89	75
21	76	79	86	76	76	79	83	76	78	83	85	66	78	83	85	57	93	96	97	79
22	56	65	74	25	56	65	74	25	64	68	69	62	69	72	75	62	84	83	84	99
23	66	70	73	56	68	73	76	56	73	74	78	59	79	78	82	76	86	84	85	99
24	54	62	70	19	56	60	67	33	63	61	72	69	76	72	78	88	86	86	88	87
25	64	72	75	46	68	76	75	50	78	81	81	70	84	86	88	76	91	90	91	99
26	58	66	58	55	66	68	57	50	71	74	70	42	76	82	77	83	89	89	87	99
27	68	79	76	61	68	75	75	61	75	83	82	50	76	84	85	44	86	90	89	50
28	64	68	67	85	68	70	78	69	73	73	72	85	81	80	79	99	88	90	89	79
29	68	76	75	53	68	75	75	50	71	75	75	58	83	81	81	88	91	90	91	99
30	66	68	69	77	66	68	70	69	75	74	78	77	76	75	77	77	88	89	87	87



Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
DEC 1	61	69	67	61	61	69	67	61	69	75	75	63	78	84	84	58	91	92	93	99
2	73	78	75	78	73	78	75	58	78	78	76	72	84	87	87	71	93	94	93	75
3	69	75	68	62	76	81	79	62	81	82	81	77	88	88	90	99	89	91	94	99
4	69	71	75	63	76	76	80	62	81	83	84	62	88	92	91	50	89	92	92	59
5	76	83	83	58	76	82	85	45	76	77	78	62	81	83	83	50	88	90	91	50
6	64	65	59	57	68	69	65	62	71	73	71	63	78	79	77	77	83	84	83	79
7	51	58	59	31	54	60	62	35	61	65	67	41	68	74	76	39	78	81	86	50
8	56	58	69	62	56	60	71	56	59	62	71	59	44	70	79	50	84	80	80	99
9	58	61	66	38	61	64	64	38	71	72	78	57	75	74	75	58	79	82	78	66
10	64	65	61	53	66	67	63	58	69	69	65	69	71	73	69	55	84	87	85	66
11	71	74	78	91	73	75	79	91	79	80	83	92	84	86	84	91	98	92	92	83
12	68	74	72	50	69	75	73	50	78	77	70	75	81	82	78	66	89	89	89	99
13	69	70	71	63	71	73	75	63	76	80	83	63	81	87	80	55	89	92	95	66
14	71	71	62	75	71	72	64	72	78	80	78	77	84	87	88	83	91	90	91	99
15	75	76	86	66	78	81	90	66	83	85	89	77	84	84	90	83	89	89	93	99
16	66	71	78	59	71	72	74	87	76	77	77	85	83	86	86	62	88	87	87	99
17	68	77	81	53	69	74	79	61	73	73	74	72	73	73	75	71	91	92	95	84
18	69	73	74	55	73	78	81	54	79	84	85	66	81	86	83	69	84	83	81	99
19	71	73	73	72	71	75	75	66	76	83	83	42	86	91	92	66	84	96	97	88
20	66	62	54	72	88	82	54	72	71	85	62	87	78	75	71	89	84	84	81	99
21	53	59	59	37	54	62	62	37	63	69	69	43	75	78	79	81	88	92	91	66
22	66	78	79	80	68	75	75	53	73	76	76	61	79	79	81	89	89	92	95	99
23	73	72	71	71	75	73	75	75	79	79	72	77	84	83	77	88	89	88	87	99
24	68	77	82	36	68	77	83	36	71	81	88	22	81	90	92	37	84	90	93	33
25	71	73	76	79	57	75	79	89	81	81	84	99	86	87	89	79	93	94	93	99
26	66	69	69	63	66	68	67	69	75	77	77	75	83	82	86	83	80	87	89	99
27	66	77	76	38	68	79	80	42	69	82	84	36	75	81	83	44	86	90	91	57
28	64	69	70	55	64	70	71	50	71	73	75	62	75	77	80	66	86	88	89	59
29	78	84	78	83	79	87	82	83	86	90	87	72	88	91	88	79	89	92	91	66
30	64	70	66	66	64	68	64	79	75	75	71	99	76	77	73	99	86	87	85	75
31	71	76	78	71	73	79	81	66	76	79	82	69	83	86	87	75	94	94	93	99

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
JAN 1	56	62	73	44	56	61	70	50	66	69	75	66	73	75	84	66	78	78	83	66
2	69	88	85	50	71	88	85	47	75	82	78	64	79	86	84	58	93	95	95	83
3	71	76	76	99	71	76	76	99	75	77	75	85	83	83	84	38	91	91	91	99
4	59	65	65	50	59	65	66	50	68	73	74	59	78	81	84	62	88	90	90	59
5	66	66	64	66	71	75	75	66	76	82	81	66	70	82	82	85	88	90	89	79
6	61	67	66	50	66	69	66	66	75	78	82	57	78	80	84	75	88	88	89	99
7	61	62	70	69	71	75	79	69	75	73	80	89	75	74	81	88	89	90	91	83
8	73	81	73	71	73	79	79	59	78	82	81	58	83	88	91	58	88	90	93	59
9	68	70	69	57	68	70	65	55	75	78	75	62	78	85	84	39	89	92	91	79
10	66	75	83	53	69	78	83	61	75	79	86	59	81	81	83	71	89	90	93	75
11	61	72	74	59	66	76	78	55	71	77	80	55	81	85	84	66	88	88	87	79
12	64	75	75	54	66	75	75	59	69	74	74	69	73	77	73	57	84	86	85	83
13	71	71	79	99	71	72	79	99	76	78	87	90	79	81	84	81	88	92	91	85
14	68	79	78	45	89	79	75	54	71	80	81	44	75	85	90	50	83	90	89	25
15	66	68	64	88	69	69	64	88	73	74	72	88	76	80	80	85	86	91	91	79
16	71	77	71	61	71	76	72	61	73	79	75	54	78	82	78	55	86	84	84	99
17	64	72	74	66	69	74	75	69	71	77	79	66	78	80	84	75	88	90	90	75
18	73	74	77	83	73	73	78	81	75	76	79	79	79	80	84	77	84	84	85	79
19	89	77	82	50	69	77	80	45	71	77	81	50	84	89	92	55	89	92	91	75
20	75	83	82	69	78	88	88	69	81	90	91	69	84	88	90	79	93	96	91	50
21	75	77	82	85	76	80	83	79	81	83	84	99	84	84	84	83	91	92	92	50
22	63	62	62	69	66	67	65	66	69	67	68	67	78	78	81	75	89	90	90	75
23	63	71	71	52	66	77	77	66	71	80	78	50	75	82	82	63	89	90	89	79
24	81	86	88	63	83	89	93	55	84	88	91	75	94	95	97	87	98	99	99	79
25	76	81	90	79	78	83	91	75	18	86	97	79	89	91	93	99	96	96	96	1
26	78	80	82	66	79	80	80	75	81	83	84	71	83	83	84	79	88	89	89	50
27	66	70	70	55	69	72	71	66	73	77	78	62	78	83	82	55	88	92	92	66
28	76	87	90	50	78	85	85	53	81	88	92	54	86	87	88	87	91	92	91	99
29	78	78	79	79	79	80	83	83	83	83	84	79	86	88	87	83	89	89	89	99
30	69	74	75	50	71	75	76	55	79	85	85	50	88	94	93	50	91	94	93	66
31	79	88	91	66	83	86	88	83	86	89	93	71	93	94	97	99	98	98	98	99

Period	Threshold = .02 inch				Threshold = .05 inch.				Threshold = .10 inch				Threshold = .20 inch				Threshold = .50 inch			
	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD	D	D/D	D/DD	D/WD
FEB 1	63	63	67	39	66	67	67	50	73	75	72	59	78	80	79	66	91	91	94	99
2	68	76	77	47	68	77	76	43	78	81	82	46	76	78	77	63	94	94	94	99
3	63	68	65	55	63	68	64	55	69	72	69	62	79	82	78	59	88	89	88	66
4	63	65	64	53	69	71	67	61	73	76	75	66	79	79	78	87	89	90	90	83
5	68	76	75	53	69	76	77	54	73	70	78	59	76	83	84	50	79	83	83	39
6	64	65	72	66	69	71	71	59	76	77	74	66	78	80	77	62	89	91	91	88
7	63	71	74	57	64	76	79	41	73	80	79	59	78	85	86	66	89	92	95	99
8	73	86	89	63	76	84	84	79	83	90	94	66	88	93	79	71	94	96	95	75
9	73	81	84	39	79	82	87	50	83	83	82	75	88	90	88	33	93	94	94	50
10	75	88	94	12	75	85	92	25	83	85	90	75	84	88	87	59	88	91	90	66
11	68	75	71	79	69	75	73	85	75	77	79	71	86	90	89	83	89	92	92	99
12	68	85	85	27	69	85	88	27	71	82	84	36	78	80	82	79	89	94	93	75
13	64	65	77	83	69	69	80	99	78	76	83	99	86	87	88	89	93	94	94	99
14	73	76	74	71	73	76	75	69	81	85	84	69	84	86	85	83	91	94	94	66
15	79	81	73	99	79	81	75	89	83	87	84	85	89	90	91	99	98	99	99	99
16	73	72	77	75	76	77	77	75	81	83	88	66	84	85	86	79	93	93	92	1
17	69	72	79	69	73	76	81	72	84	81	78	89	88	86	84	99	89	89	89	99
18	78	80	81	83	79	81	82	81	79	82	82	66	86	88	90	71	88	90	91	66
19	75	78	76	62	78	81	80	82	78	81	80	66	81	80	82	83	91	92	93	79
20	66	71	70	50	66	70	69	44	73	76	76	55	81	93	80	69	84	87	85	50
21	68	69	68	69	69	69	69	71	73	75	72	72	79	79	80	75	84	89	89	71
22	73	82	85	58	76	85	85	58	81	84	84	81	89	93	97	69	91	96	97	57
23	66	77	76	57	68	76	77	66	71	73	75	57	76	75	79	99	83	83	83	99
24	73	82	85	50	73	82	85	54	75	81	83	61	81	89	92	53	93	95	97	77
25	69	81	84	57	71	81	85	57	75	82	88	50	79	87	92	39	89	91	93	50
26	79	85	83	62	79	86	83	62	84	88	89	75	83	86	90	83	86	88	93	79
27	71	75	86	83	73	75	83	83	79	82	89	79	83	86	90	83	86	88	93	79
28	64	69	69	41	69	75	75	50	83	83	80	88	83	87	86	99	93	94	93	99